Math 62 13.1 - 3rd Review of Sequences & Series

Math 72 11.3 - 2nd

Review: Sequences and Series

Arithmetic sequence:

- Add (or subtract = add negative) the same number *d* each time
- $a_n = a_1 + d(n-1)$

Geometric sequence

- Multiply (or divide = multiply by a fraction) the same number *r* each time
- $a_n = a_1 \cdot r^{n-1}$

Practice

Write the general term for each sequence.

1) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

2) $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

3) $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

4) $2, -1, -4, -7, -10$

Write in summation notation.

5) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

6) $2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

7) $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

8) $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

9) Find a_{100} when $a_n = \frac{(-1)^{n+1} \cdot n^2}{4}$

10) Evaluate

$$\sum_{n=1}^{6} 3 \cdot 2^n$$

11) Evaluate S_3 when $a_n = 7n - 1$

12) Write the general term: $3, -2, -7, -12, -17$

13) Write in summation notation: $10 + 2 + \frac{2}{5} + \frac{2}{25}$

14) Find the first six terms of $a_n = 7\left(-\frac{2}{5}\right)^{n+1}$

Write the general term for each sequence.

(1) $2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$

↳ mult by $\frac{1}{3}$ each time = Geometric sequence

$a_n = a_1 \cdot r^{n-1}$ $\quad a_1 = 2$, $r = \frac{1}{3}$

$$\boxed{a_n = 2 \cdot \left(\frac{1}{3}\right)^{n-1}}$$

(2) $2, \frac{8}{3}, \frac{10}{3}, 4, \frac{14}{3}$

notice

$2 = \frac{6}{3}$ and $4 = \frac{12}{3}$

so the sequence is $\frac{6}{3}, \frac{8}{3}, \frac{10}{3}, \frac{12}{3}, \frac{14}{3}$

↳ add $\frac{2}{3}$ each time = Arithmetic sequence

$a_n = d(n-1) + a_1$ $\quad a_1 = 2$, $d = \frac{2}{3}$

$a_n = \frac{2}{3}(n-1) + 2$

$= \frac{2}{3}n - \frac{2}{3} + 2$ dist

$$\boxed{a_n = \frac{2}{3}n + \frac{4}{3}}$$

(3) $6, -3, \frac{3}{2}, -\frac{3}{4}, \frac{3}{8}$

notice mult by $\frac{1}{2}$ each time = Geometric sequence

$a_n = a_1 \cdot r^{n-1}$ $\quad a_1 = 6$, $r = \frac{1}{2}$

$$\boxed{a_n = 6\left(\frac{1}{2}\right)^{n-1}}$$

④ $2, -1, -4, -7, -10$

Notice add 3 each time = Arithmetic Sequence

$a_n = d(n-1) + a_1$ $\quad a_1 = 2$, $d = 3$

$a_n = 3(n-1) + 2$

$a_n = 3n - 3 + 2$

$\boxed{a_n = 3n - 1}$

Write in Summation notation.

⑤ $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27}$

Same sequence as ①, only added, so it's a series

$\boxed{\sum_{n=1}^{4} 2\left(\frac{1}{3}\right)^{n-1}}$ 4 terms

⑥ $2 + \frac{8}{3} + \frac{10}{3} + 4 + \frac{14}{3} + \frac{16}{3}$

Same sequence as ② 6 terms

$\boxed{\sum_{n=1}^{6} \left(\frac{2}{3}n + \frac{4}{3}\right)}$

⑦ $6 - 3 + \frac{3}{2} - \frac{3}{4} + \frac{3}{8}$

Same sequence as ③ 5 terms

$\boxed{\sum_{n=1}^{5} 6\left(\frac{1}{2}\right)^{n-1}}$

⑧ $2 - 1 - 4 - 7 - 10 - 13 - 16 - 19 - 21$

Same sequence as ④ 9 terms

$\boxed{\sum_{n=1}^{9} (3n-1)}$

(9) Find a_{100} when $a_n = \frac{(-1)^{n+1} \cdot n^2}{4}$

substitute $n = 100$

$$a_{100} = \frac{(-1)^{100+1} \cdot (100)^2}{4}$$

$$= \frac{(-1)^{101} \cdot 10000}{4}$$ add power ⇒ negative

$$= \boxed{-2500}$$

(10) Evaluate $\sum_{n=1}^{6} 3 \cdot 2^n$

sigma means add
6 terms
Subst $n=1, \ldots n=6$
add results

$$= 3 \cdot 2^1 + 3 \cdot 2^2 + 3 \cdot 2^3 + 3 \cdot 2^4 + 3 \cdot 2^5 + 3 \cdot 2^6$$

$$= 6 + 12 + 24 + 48 + 96 + 192$$

$$= \boxed{378}$$

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(12) Write the general term: $3, -2, -7, -12, -17$

subtract 5 each time
= add (-5) each time
= Arithmetic sequence

$a_n = d(n-1) + a_1$ $\quad a_1 = 3$, $d = -5$

$a_n = -5(n-1) + 3$

$= -5n + 5 + 3$

$\boxed{a_n = -5n + 8}$

(11) Evaluate S_3 when $a_n = 7n-1$

$$S_3 = a_1 + a_2 + a_3$$
$$= (7(1)-1) + (7(2)-1) + (7(3)-1)$$
$$= 6 + 13 + 20$$
$$= \boxed{39}$$

(12) previous page

(13) Write in summation notation $10 + 2 + \frac{2}{5} + \frac{2}{25}$

notice: divide by 5 each time
= mult by $\frac{1}{5}$ each time
⇒ Geometric series

$a_n = a_1 \cdot r^{n-1}$ $\quad a_1 = 10$, $r = \frac{1}{5}$

$a_n = 10\left(\frac{1}{5}\right)^{n-1}$

4 terms

$$\sum_{n=1}^{4} 10\left(\frac{1}{5}\right)^{n-1}$$

(14) Find the first six terms of $a_n = 7\left(-\frac{2}{5}\right)^{n+1}$

$n=1$ $\quad 7\left(-\frac{2}{5}\right)^{1+1} = 7\left(-\frac{2}{5}\right)^2 = 7\left(\frac{4}{25}\right) = \frac{28}{25}$

$n=2$ $\quad 7\left(-\frac{2}{5}\right)^{2+1} = 7\left(-\frac{2}{5}\right)^3 = 7\left(\frac{-8}{125}\right) = \frac{-56}{125}$

$n=3$ $\quad 7\left(-\frac{2}{5}\right)^{3+1} = 7\left(-\frac{2}{5}\right)^4 = 7\left(\frac{16}{625}\right) = \frac{112}{625}$

$n=4$ $\quad 7\left(-\frac{2}{5}\right)^5 = \frac{-224}{3125}$

$n=5$ $\quad 7\left(-\frac{2}{5}\right)^6 = \frac{+448}{15\,625}$

$n=6$ $\quad 7\left(-\frac{2}{5}\right)^7 = \frac{-896}{78125}$

MATH ▸ frac may fail you!
Mult num × (-2)
denom × (5)

$\frac{28}{25}, \frac{-56}{125}, \frac{112}{625}, \frac{-224}{3125}, \frac{448}{15625}, \frac{-896}{78125}$

Math 70 Extra

Find the general term a_n of a sequence given the first several terms of the sequence.

Hints

Look for counting numbers

(3) $1, 2, 3, 4, 5 \Rightarrow a_n = n$

Look for reciprocals

(4) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \Rightarrow a_n = \frac{1}{n}$

Look for multiples

(5) $2, 4, 6, 8, 10 \Rightarrow a_n = 2n$

(6) $3, 6, 9, 12, 15 \Rightarrow a_n = 3n$

(7) $-4, -8, -12, -16, -20 \Rightarrow a_n = -4n$

(8) $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \frac{1}{25} \Rightarrow a_n = \frac{1}{5n}$

Look for powers

(9) $2, 4, 8, 16, 32 \Rightarrow a_n = 2^n$

(10) $3, 9, 27, 81, 243 \Rightarrow a_n = 3^n$

(11) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \Rightarrow a_n = \frac{1}{2^n}$

Special power is alternating signs:

(12) $-1, +1, -1, +1, -1 \Rightarrow a_n = (-1)^n$

Look for squares

(13) $1, 4, 9, 16, 25 \Rightarrow a_n = n^2$

Look for offsets

(14) $0, 1, 2, 3, 4 \Rightarrow a_n = n-1$

(15) $6, 9, 12, 15, 18 \Rightarrow a_n = 3(n+1)$

(16) $1, 2, 4, 8, 16 \Rightarrow a_n = 2^{n-1}$

(17) $3, 6, 11, 18, 27 \Rightarrow a_n = n^2+2$

Then look for combinations of these!

Math 70 Extra

Find the indicated term for each sequence whose general term is given.

(18) $a_n = \frac{n-4}{(-2)^n}$; a_6

substitute $n=6$ into the general term

$a_6 = \frac{6-4}{(-2)^6} = \frac{2}{64} = \boxed{\frac{1}{32}}$

(19) $a_n = 8-n^2$; a_{20}

$n=20 \quad a_{20} = 8-20^2 = \boxed{-392}$

(20) $a_n = \frac{(-1)^n}{2n}$; a_{100}

$n=100 \quad a_{100} = \frac{(-1)^{100}}{2(100)} = \boxed{\frac{1}{200}}$

* Use even/odd powers!

(21) $a_n = \frac{n+3}{n+4}$; a_8

$n=8 \quad a_8 = \frac{8+3}{8+4} = \boxed{\frac{11}{12}}$

(22) $a_n = 5^{n+1}$; a_3

$n=3 \quad a_3 = 5^{3+1} = 5^4 = \boxed{625}$

(23) $a_n = \frac{n}{n+4}$; a_{24}

$n=24 \quad a_{24} = \frac{24}{24+4} = \frac{24}{28} = \boxed{\frac{6}{7}}$

(24) $a_n = 100-7n$; a_{50}

$n=50 \quad a_{50} = 100-7(50) = \boxed{-250}$

(25) $a_n = -n^2$; a_{15}

$n=15 \quad a_{15} = -(15)^2 = \boxed{-225}$

Math 70 Extra Practice

To practice writing numerical terms, cover the right side of the page and use the general term to find the first four terms.

To practice finding general terms, cover the left side of the page and use the four terms on the right to find a general term.

(26) $a_n = -7n+2$

(27) $a_n = -4(2)^{n+1}$ or $-16(2^{n-1})$ or -2^{n+3}

(28) $a_n = -3n^2 - 1$

(29) $a_n = 9n - 1$

(30) $a_n = 7n + 3$

(31) $a_n = -4n - 5$

(32) $a_n = -6n + 9$

(33) $a_n = 3\left(\frac{1}{2}\right)^n$

(34) $a_n = 3\left(\frac{2}{5}\right)^n$

(35) $a_n = 5\left(\frac{2}{3}\right)^{n-1}$

(36) $a_n = n^2$

(37) $a_n = n^2 - 3$

(38) $a_n = n^2 + 4$

(39) $a_n = 2n^2$

(26) $-5, -12, -19, -26$

(27) $-16, -32, -64, -128$

(28) $-3, -12, -27, -48$

(29) $8, 17, 26, 35$

(30) $10, 17, 24, 31$

(31) $-9, -13, -17, -21$

(32) $3, -3, -9, -15$

(33) $\frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}$

(34) $\frac{6}{5}, \frac{12}{25}, \frac{24}{125}, \frac{48}{625}, \frac{96}{3125}$

(35) $5, \frac{10}{3}, \frac{20}{9}, \frac{40}{27}$

(36) $1, 4, 9, 16$

(37) $-2, 1, 6, 13$

(38) $5, 8, 13, 20$

(39) $2, 8, 18, 32$